mission on the Teaching of Mathematics, and Section IV, at their first Cambridge Meeting, express regret at your absence and best wishes for your recovery.'

"The Commission was organized for the purpose of reporting upon the present status of the teaching of mathematics in the various countries of the world. Special sub-committees have also been appointed from time to time, to consider questions of international rather than merely national interest. About one hundred and fifty reports on the work done in the various countries have been prepared, and at least fifty more are in contemplation. A world-wide interest in the improvement of mathematical teaching has been awakened, and the influence of the movement is certain to be very far reaching. Ten countries have completed the task set for themselves. In chronological order of completion these countries are Sweden, Holland, France, Switzerland, Austria, Japan, the United States of America, the British Isles, Hungary, and Denmark. In process of publication is the monumental work of Germany, with twenty-seven out of thirty-six reports already printed, and the reports of Italy, Roumania, Spain, and Russia. In contemplation are the reports of Greece, Norway, Australia, Portugal, Servia, and doubtless of several other countries.

"As to the future work of the Commission, the Central Committee earnestly desires that it be authorized to see to the completion of the reports. It is therefore very desirable that it be continued in power, both for this purpose and for the consideration of certain questions of great international significance. Such topics as the proper training of engineers, of calculus in the secondary schools, of the general value of intuition in the teaching of mathematics, of the training of teachers, and of the educational (cultural, disciplinary, non-technical) value of mathematics, may properly occupy the attention of the Commission in the next four years. Special conferences having already been held, at Bruxelles and Milano, it is proposed, if the Committee is continued in power, to hold others between now and the time of the meeting of the Congress in 1916, if that shall be the date. Possibly such conferences may be held in France in 1914, in Germany in 1915, and in Stockholm in 1916.

"It is also hoped that each country will prepare a summary of the large features of the reports of other countries, to the end that the work that has been accomplished may have its full effect. It is further hoped that the various countries will continue the financial support that has been given to the Central Committee in the past.

"A word should be said at this time in memory of those distinguished teachers who have been connected with the movement, but who have been called from their labors to solve the Great Problem. Soon after the last Congress adjourned, Professor Vailati of Rome, a distinguished writer and an accomplished scholar, passed away. Scarcely in his full prime of life, his loss is felt not by Italy alone, but by all who appreciate scholarship

and high educational standards. Professor Bovey, president of the Imperial Technical College at South Kensington, and who was charged with the labor of reporting for Canada, has also been called from us. In his death the world lost a scholar and an administrator of prominence. And as he was planning to attend this Congress, four weeks ago today, Geheimrath Professor P. Treutlein of Carlsruhe, passed suddenly away. In his death Germany lost one of her foremost educators, and the International Commission, one of its best supporters.

"The Central Committee has consulted with the Committee on Organization and it has been decided that the first set of reports shall be presented to the Library of the University of Cambridge, a second set to our official hosts, the Cambridge Philosophical Society, and a third set to that great world-library, the Library of the British Museum."

The General Secretary of the Commission next made a statement as to the work of the Central Committee, and submitted its collected publications.

Thereupon the reports of the various countries were formally submitted to the Congress. The countries were called in alphabetical order in the French language, and the following members of the Commission presented the reports, with a brief oral description, and a longer written statement, which will be published in the Proceedings of the Congress, and in the official organ of the Commission, *L'enseignement mathématique*. The statement accompanying the American report will be published in *School Science and Mathematics*, the official American organ.

Germany, Prof. A. Gutzmer (Halle); Austra, Prof. E. Czuber (Vienna); Belgium, Principal E. Clevers (Ghent); Denmark, Prof. H. Fehr; Spain, Prof. Toledo (Madrid); United States, Prof. J. W. A. Young (Chicago); France, Prof. C. Bourlet (Paris); Greece, Prof. H. Fehr; Holland, Prof. J. Cardinaal (Delft); Hungary, Prof. E. Beke (Buda-Pesth); British Isles, Prof. C. S. Jackson (Woolwich); Italy, Prof. G. Castelnuovo (Rome); Japan, Prof. R. Fujisawa (Tokio); Norway, Prof. M. Alfsen (Christiania); Portugal, Prof. F. J. Teixeira (Oporto); Roumania, Prof. G. Tzitzeica (Bucharest); Russia, Prof. H. Fehr; Sweden, Prof. H. Fehr; Switzerland, Prof. H. Fehr (Geneva).

Also the following associated countries: Brazil, Prof. E. de B. R. Gabaglia (Rio de Janeiro); Servia, Prof. M. Petrovitch (Belgrade).

At the second joint session of Section IVb and the International Commission, the report of sub-commission B, on "The mathematical education of the physicist in the university," was presented by Professor C. Runge, and followed by a lively discussion.

At the last joint session of Section IVb and the International Commission, C. Goldziher presented a report on the work done by David Eugene Smith and himself towards preparing a bibliography of works on the teaching of mathematics, published since 1900. (This bibliography is about to be

published by the United States Bureau of Education and can be obtained from the Bureau on request.) Upon motion of Professor Smith the following resolution was passed:

"*Resolved*, That Section IVb of the International Congress of Mathematicians, assembled at Cambridge, expresses its thanks to the Honorable the United States Commissioner of Education for his great interest in publishing, for free distribution, the recent bibliography on the teaching of mathematics (1900-1912), and the hope that it may, through his good offices, be brought to completion to the year 1915, with such additions to the present list as may seem desirable."

David Eugene Smith then presented the Report of Sub-commission A on "Intuition and experiment in mathematical teaching in secondary schools." The presentation of the report was followed by an extended discussion. This report will be published in the various official organs named above.

In the other sessions of Section IVb, the following papers were presented:

Whitehead, A. N. "The principles of mathematics in relation to elementary teaching."

Suppandschitch, R. "Le raisonnement logique dans l'enseignement mathématique universitaire et secondaire."

Hill, M. J. M. "The teaching of the theory of proportion."

Hatzidakis, N. "Systematische Recreationsmathematik in den mittleren Schulen."

Gérardin, A. "Sur quelques nouvelles machines algébriques."

Carson, G. St. L. "The place of deduction in elementary mechanics."

Nunn, T. P. "The proper scope and method of instruction in the calculus in schools."

It was not possible to secure brief abstracts of the above papers for incorporation in this report. The papers will be published in full in the Proceedings of the Congress, and elsewhere.

This account of the Congress would be lamentably incomplete without brief mention of its social side. First of all, the four official receptions, that by Sir G. H. Darwin, President of the Cambridge Philosophical Society, in St. John's College, on Wednesday evening; that by Lord Rayleigh, Chancellor of the University, in the Fitzwilliam Museum, on Friday evening; that by the President of the Congress in Christ's College, on Sunday afternoon; and finally that by the Master and Fellows of Trinity College on Monday evening. These brilliant functions, each in a unique setting with a charm all its own, will remain in memory as pictures not to be forgotten. Sunday was fittingly closed with an organ recital in King's College Chapel, the most lustrous of all of Cambridge's architectural gems. Visits to the Observatory, and to the Cambridge Scientific works, with attendant teas, excursions to Ely, to Oxford, and to Hatfield House, were temptations that caused the de-

votion to his science of more than one mathematician temporarily to waver. Besides all this an able and enterprising committee of ladies had prepared for the ladies of the Congress a most attractive series of visits to the Colleges, of drives and teas in Cambridge and its environs, and of excursions to various points of interest that seemed to leave no moment without something tempting to do. Whether viewed from the social side or from the mathematical side, the Congress must be pronounced a complete success.

THE WORD "RADIAN."

By A. R. CRATHORNE, University of Illinois.

Few terms in elementary mathematics are of such recent origin that the time and place of their first introduction into the literature can be determined. The word "radian" seems to be an exception to this statement, however, for its history goes back some forty years only. One of the originators of the word is still living.

When the "R" volume of Murray's "New English Dictionary" appeared, it gave under the definition of "radian" the date 1879, and a reference to the *Treatise on Natural Philosophy*, by Thomson and Tait, from which one inferred that here was the first use of the word. The derivation was given as "radius+an." This statement in the dictionary called forth a letter to *Nature* (Vol. 83, p. 156) from Professor Thomas Muir of Cape Town, South Africa, who claimed to have used the term in his classes at St. Andrew's University as early as 1869. At that time he was hesitating between the three words "rad," "radial" and "radian" with leanings towards the monysyllable. In 1874 after discussing the matter with the late Professor James Thompson and with Alexander Ellis, he finally adopted the term "radian," considering it as a contraction of "radial angle."

In answer to the letter of Professor Muir, Mr. James Thomson, son of Professor James Thomson, wrote a letter to *Nature* (Vol. 83, p. 217), in which he pointed out that the word had been used in 1871 by his father, before the latter knew Dr. Muir, and that on June 5, 1873, the word was printed in the examination questions at Queen's College, Belfast. These questions were published in the college calendar.

Two more letters from Mr. Thomson and Dr. Muir on the subject appear in the same volume of *Nature*, pp. 459, 460, from which it is seen that the term "radian" was used independently for some years by Professor Muir and Professor James Thomson, but that the first printed word was in the above mentioned examination papers. Professor Muir also originated the physical term "therm" in 1870 (see *Nature*, Vol. I, p. 606).

HISTORICAL NOTE ON THE GRAPHIC REPRESENTATION OF IMAGINARIES BEFORE THE TIME OF WESSEL.

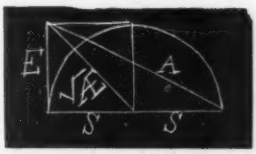
By FLORIAN CAJORI, Colorado College.

John Wallis's attempts at graphic representation of imaginaries have been described by H. Hankel,* W. W. Beman,† D. E. Smith,‡ and more fully by G. Eneström.§ They refer to Wallis's *Treatise of Algebra*, London, 1685, pp. 264-273, but none of them mention Wallis's earlier discussion of this subject in a letter to Collins, May 6, 1673, where he suggests a construction a little different from any of the constructions found in his *Algebra*. This letter was written three or four years before the manuscript of his *Algebra* was ready for print. Wallis says in the letter:‖

"This imaginable root in a quadratic equation I have had thoughts long since of designing geometrically, and have had several projects to that purpose. One of them was this: Supposing a quadratic equation $2SA-A^2=Æ$, or (which is equivalent) $A^2-2SA+Æ=0$. If $S\ (=\frac{A+E}{2})$ be bigger than $\sqrt{(Æ)}$; that is $S^2>Æ$, the roots are $S\pm\sqrt{(S^2-Æ)}=\begin{cases}A\\E,\end{cases}$ putting . . . $S=\frac{1}{2}Z=\frac{A+E}{2}$ and . . . $V=\frac{1}{2}X=\frac{A-E}{2}$,

where $V\ [=\sqrt{(S^2-Æ)}]$, added to and taken from S, yields $S+V=A$, $S-V=E$, that is, [the roots are] $S\pm\sqrt{(+V^2)}$.

"But if $Æ$ be bigger than S^2, the roots are $S\pm\sqrt{(S^2-Æ)}$ $(=S\pm\sqrt{(-V^2)})$, where $\sqrt{(Æ)}$, which was the sine, now becomes the secant, and V, that was the cosine, is now the tangent. For $S^2\sim Æ=V^2$, the difference of the planes S^2 and $Æ$, the greater is to be expressed by the hypotenuse, and the less by the perpendicular."



Evidently, S and $Æ$ are here always positive, hence this is not a general construction of the roots of the quadratic equation.

In both figures the lines E and A represent the roots of the quadratic. In both, the line E extends from the left end of the diameter to the terminal of the line V; the line A begins where E ended and extends to the right end of the diameter. Thus the analogy in the construction of real roots in the first figure and of complex roots in the second figure is complete. More-

* Hankel. *Compleze Zahlen*, Leipzig, 1867, pp. 81, 82.
† *Proceedings of the American Association for the Advancement of Science*, Vol. 46, 1897, pp. 35, 36.
‡ Merriman and Woodward, *Higher Mathematics*, 1898, pp. 515, 516.
§ *Bibliotheca Mathematica*, 3rd S., Vol. 7, pp. 263-269.
‖ S. J. Rigaud, *Correspondence of Scientific Men of the Seventeenth Century*, Vol. 2, Oxford, 1841, p. 578.

over there is an attempt to secure vector addition. We have $E+A=2S$, also $a+ib+a+ib=2a+2ib$; but we do not have $a+ib+c+id=(a+c)+i(b+d)$. This method of geometric representation of imaginaries labors under another fatal defect, that of representing conjugate roots by lines of different lengths. While this method is less interesting than some others found later in Wallis's *Algebra*, it has one point of superiority over them in being perfectly determinate. Eneström has shown that some of Wallis's geometric notions permit the vector standing for a complex root to take any one of an indefinite number of different directions.* It will be noticed that the present method fails in case of the pure imaginary $\sqrt{(-1)}$, since for that case the radius S of the circle vanishes.

It is well known now that Kühn, in 1750, did not attempt a geometric picture of an imaginary, but simply tried to interpret a negative plane.† He was anticipated by Wallis, who touches upon this point not only in his *Algebra*, but also in the letter of 1673, from which we have been quoting. Wallis says in the letter:

"I was of opinion from the first, that a negative plane may as well be admitted in algebra as a negative length, both being in nature equally impossible; for there can no more be a line less than nothing than a plane less than nothing, both being but imaginable; and if we suppose such a negative square, we may as well suppose it to have a side, not indeed an affirmative, or a negative length, but a supposed mean proportional between a negative and positive thus designable, $\sqrt{(-n)}$, or rather $\sqrt{(-n^2)}$, that is, $\sqrt{(+nx^x-n)}$, a mean proportional between $+n$ and $-n$."

No doubt Wallis touched upon geometric notions on imaginaries in other letters. There is a reference to this subject in one of September 11, 1676.‡ Before this, Collins expressed himself in a letter to J. Gregory, October 19, 1675, as follows:§

"I nothing doubt but the roots of such negative squares, etc., denote an impossibility; as for instance Dr. Wallis, in his first tome, assumes the two legs of a triangle, 2 and 1, to be less than the base 4, and that to show that algebra might be fallacious to a tire, and really finds the segments of the base well; but had he proceeded further he would have found the perpendicular to have been $\sqrt{(-105)}$, which had minifested the impossibility."

A few years ago Felix Müller made the statement that Euler had given a geometric representation of the imaginary by means of a circle.‖ Eneström has explained the construction¶ and pointed out that it is the same as one given a hundred years earlier by Wallis, and is in fact trivial. It is quite possible that Euler, like Wallis, may have entertained different

* *Bibliotheca Mathematica*, 3rd S, Vol. 7, p. 266.

† Eneström in *Bibliotheca Mathematica*, 3rd S, Vol. 7, pp. 263-269.

‡ Rigaud, *op. cit.*, Vol. 2, p. 594.

§ Rigaud, *op. cit.*, Vol. 2, p. 278.

‖ *Festschr. z. Feier d. 200. Geburtstages L. Eulers*, Leipzig und Berlin, 1907, p. 96; *Opusc. analyt.*, Vol. 2, pp. 76-90.

¶ *Bibliotheca Mathematica*, 3rd S., Vol. 9, 1908-1909, p. 182.

schemes. The query has arisen in the mind of the writer whether not only Euler but also Charles Walmesley had the Gaussian representation in mind when writing the passages we are about to quote. In Euler's great article, *De la controverse entre Leibnitz et Bernoulli*, etc., on the logarithms of complex numbers,* it is explained under "Probleme 4" how to find the anti-logarithm, when the logarithm $g\sqrt{(-1)}$ is given. Euler says:

". . . pour le trouver on n'a qu'a prendre un arc de cercle=g, le rayon etant=1 et ayant cherche son sinus et cosinus, le nombre cherche sera $x=\cos g+\sqrt{(-1)}\sin g$."

Much the same language is used by Euler in another article in the same volume of the Berlin memoirs (p. 278), *Sur les racines imaginaires des equations.* Six years later Charles Walmesley, a Roman Catholic prelate, wrote an article on logarithms.† To find the logarithm of $a+b\sqrt{(-1)}$, he changes it to the form $(a^2+b^2)^{-\frac{1}{2}}(y+u\sqrt{(-1)})$, and says, "il est clair qu'on doit prendre l'arc de cercle dont le sinus est u, le cosinus y," etc.

Did Walmesley and Euler, in connection with $y+ui$, carry in their minds the geometric picture of the lines that were generally used at that time and long before to represent u the sine and y the cosine, the two lines being perpendicular to each other? In other words, did Euler and Walmesley have in mind the Wessel-Argand diagram? There is danger of reading into passages ideas which the authors had not entertained. Hence we must wait for more careful studies of other articles by the same authors, before a final conclusion can be reached.

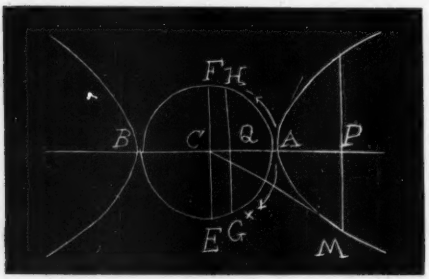
Another conjecture presents itself. It is asserted that Gauss arrived at the Wessel-Argand diagram independently of Wessel and Argand. We know that Gauss in his doctor's dissertation of 1799 passes in critical review the eighteenth century proofs of the theorem that every equation has a root, including Euler's proof‡ contained in the article *Sur les racines imaginaires des equations*, named above. Our conjecture is that the diagram suggested itself to Gauss upon the reading of the passages in Euler, quoted above.

That the Wessel-Argand diagram was probably in the minds of certain eighteenth century mathematicians before the time of Wessel appears also from a remark of Cauchy,§ to the effect that "a modest scholar," Henri Dominique Truel, as early as the year 1786 represented imaginary quantities upon a line perpendicular to the line for real quantities. We are not aware that Truel ever published his results.

The most interesting and valuable graphic representation of imaginaries before Wessel has never been described by historians of mathematics. It is accomplished by means of a circle and an equilateral hyperbola. Wallis in his *Algebra* suggests various constructions of imaginaries. One of them represents $\sqrt{(1-x^2)}$ as the ordinate of the circle $x^2+y^2=1$ when $x<1$, and

* *Histoire de l'academie r. d. sciences et b. l.*, année 1749, Berlin, 1851, pp. 139-179.
† *Histoire d. l'academie r. d. sciences et b. l.*, année 1755, Berlin, 1757, p. 397.
‡ Ostwald's *Klassiker der Exakt. Wiss.*, No. 14, Leipzig, 1890, pp. 13-21.
§ Cauchy, *Exercises d'analyse et de phys. math.*, T. iv, 1847, p. 157.

as the ordinate of the hyperbola $x^2-y^2=1$ when $x>1$. During the eighteenth century one encounters not infrequently statements to the effect that real (imaginary) arcs of the circle are imaginary (real) arcs of the hyperbola.* These ideas led W. J. G. Karsten in 1768† to the invention of a diagram displaying the infinitely many logarithms of a real or a complex number. He remarks that all ordinates of the circle $x^2+z^2=1$ are imaginary ordinates of the hyperbola $x^2-y^2=1$, where $y=\sqrt{(-1)}z$, that therefore the circle may be regarded as an imaginary part of the hyperbola, and *vice versa*. Consider each of the four arcs coming together at A, or at B, as the continuation of each of the other three arcs. An arc AG for the circle may be considered as an imaginary arc of the hyperbola. Between any two points M and G exist numberless different arcs, MAG, $MAGEBFG$, or in general, $MAG+2\lambda\pi$, and also $MAHBG+2\lambda\pi$, where $\lambda=0, 1, 2, \ldots$ This is true even if M, or G, or both, coincide with A. To any abscissa x there belong therefore not only numberless arcs, but also numberless corresponding sectors. By the Calculus, double the area of a hyperbolic sector corresponding to the abscissa x is$=\log(x+y)=\sqrt{(-1)}.\text{arc.cos}x$, the sector being assumed zero when $x=1$. If $x>1$, say $x=CP$, then $\log(x+y)$ gives double the sectorial areas corresponding to the arcs $AM\pm\lambda\,(AGBFA)$, that is, $AM\pm2\lambda\pi\sqrt{(-1)}$. Only the sector ACM is real. If $x<1$, then $x+y$ is imaginary. Let $x=CQ$, then $\log(x+y)$ equals double the sectorial areas whose arcs are $AG\pm2\lambda\pi\sqrt{(-1)}$, or $AFBG\pm2\lambda\pi\sqrt{(-1)}$. All of these arcs and sectors are imaginary. From this we see that $\log(-1)$ is represented by double the areas of the sectors belonging to arcs $AEB+\lambda\,(BFAEB)$, or also $AFB+\lambda\,(BEAFB)$.



This graph is in accordance with Euler's formula $\log(-1)=(2\lambda+1)\,\pi\sqrt{(-1)}$. When $x=0$ and $\lambda=0$, we get the particular logarithm of $\log\sqrt{(-1)}$ which is represented by twice ACE. Here then Karsten gives a graphic representation of the well-known Bernoulli-Euler expression $\log\sqrt{(-1)}=\frac{\pi}{2}\sqrt{(-1)}$. In 1842 DeMorgan remarked that "not many years since, [it] was one of the mysteries of analysis," and he apparently believed that he himself had given the earliest geometric interpretation of it.‡ Karsten's diagram gives a geometric picture of the natural logarithms of any real or imaginary number. He also considers the possibility of a graphic representation by using, in place of sec-

* See for instance, Vincenzo Riccati *Sopra logaritmi dei numeri negativi lettere cinque*, Modena, 1779, p. 66. John Playfair in *Phil. Trans.*, Vol. 68, year 1778, Pt. i, pp. 318-343.

† "Abhandlung von den Logarithmen verneinter Grössen," 1. und 2. Abtheil., *Abh. Münch. Akad.* V, 1768

‡ *Transactions of the Cambridge Philosophical Society*, Vol. 7, 1842, p. 294.

tors, the hyperbolic trapezoids (the parallel sides being parallel to one of the asymptotes, the other two sides being the hyperbolic arc and the other asymptote), but he finds this less general and less convenient. Nowhere, either in eighteenth century or nineteenth century authors have we been able to find a reference to Karsten's geometric construction of imaginary logarithms.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

374. Proposed by H. PRIME, Boston, Massachusetts.

Divide an angle of 30° into two parts so that the product of the third and fourth powers of their sines (or cosines) shall be a maximum. To be solved without using the methods of calculus. [From *The Maine Farmers' Almanac*, 1912.]

Solution by H. E. TREFETHEN, Colby College.

Let x and $30°-x$ be the two parts. $\sin^4 x \sin^3(30°-x)$ = maximum, when $\sin^4 x \sin(30°-x)$ = maximum, or when $\sin^4 x \cos x - \sin^5 x \sqrt{3}$ = maximum. Put $\sin^4 x = y$, $\cos x = (1-y^6)^{\frac{1}{2}}$; and then $y^4(1-y^6)^{\frac{1}{2}} - y^7\sqrt{3} = m$, whence

$$y^{14} - y^8/4 + my^7\sqrt{3}/2 + m^2/4 = 0 \ldots (1).$$

Let c be a value of y that renders m a maximum. Then the first member of (1) must be exactly divisible *twice* by $y-c$, since a maximum or minimum corresponds to two equal roots. The quotient is readily written down by the synthetic process. The first remainder $= c^{14} - c^8/4 + mc^7\sqrt{3}/2 + m^2/4 = 0$, since the division must be exact. Also the second remainder $= 14c^{13} - 2c^7 + 7mc^6\sqrt{3}/2 = 0$.

Eliminating m from the last two equations we get $196c^{14} - 203c^8 + 16c^2 = 0$. Whence $c^2 = 0$ corresponding to a minimum, and $196c^{12} - 203c^6 + 16 = 0$, $c^6 = y^6 = \sin^2 x = 4.813227/56$, or $53.186773/56$. The former gives $x = 17° 2' 52.9''$; the latter ($= \cos^2 x$) gives $x = 12° 57' 7.1''$, and $30° - x = 12° 57' 7.1''$ or $17° 2' 52.9''$.

It is to be noted that this method for determining maxima and minima can be applied to *any algebraic* expression to which the methods of calculus are applicable.

Also solved by J. Scheffer and A. H. Holmes.

375. Proposed by S. LEFSCHETZ, University of Nebraska.

Prove that $\frac{e}{2m+2} < e-(1+\frac{1}{m})^m < \frac{e}{2m+1}$. [Schlömilch.]

Solution by H. E. TREFETHEN, Colby College.

Dividing by e and subtracting each member from unity, we deduce

$$1/(1+1/2m) < (1+1/m)^m/e < (1+1/2m)/(1+1/m) \ldots (1).$$

(i) Let n be positive, and put $m=-n$. Then (1) becomes

$$1/(1-1/2n) < (1-1/n)^{-n}/e < (1-1/2n)/(1-1/n) \ldots (2).$$

$-\log(1-1/2n) < -n\log(1-1/n) - 1 < \log(1-1/2n) - \log(1-1/n)$ or

$$1/2n+1/2^2.2n^2+1/2^3.3n^3+\ldots+1/2^r rn^r$$
$$<1/2n+1/3n^2+1/4n^3+\ldots+1/(r+1)n^r$$
$$<1/2n+(2^2-1)/2^r.2n^2+(2^3-1)/2^3.3n^3+\ldots+(2^r-1)/2^r rn^r.$$

These series all converge when $n>1$. We also have

$$1/2^r rn^r < 1/(r+1)n^r < (2^r-1)/2^r rn^r.$$
$$2^r > (r+1)/r > 2^r/(2^r-1, \quad 2^r-1 > 1/r > 1/(2^r-1).$$

For $2^r-1>r>1/r$ when $r \geq 2$ (r being a positive integer).

Thus the given relations are established for the case when $n>1$, $-n<-1$, that is when $m<-1$.

(ii) In (1) put $m=n-1$. Then after multiplying by $n/(n-1)$ we find

$$1/(1-1/2n) < (1-1/n)^{-n}/e < (1-1/2n)/(1-1/n).$$

This is the same as (2) in (i) and the case is proved when $n>1$, $n-1>0$, that is when $m>0$.

(iii) If $m=0$, the given expressions become $e/2<e-1<e$ and the case is proved for $m=0$.

(iv) But in the interval $0>m \geq -1$ there are various results for different values of m.

(a) For such values of m as render the given functions real and finite, the given inequalities are true if $0>m>1/2$, but must be reversed if $-1/2>m>-1$.

(b) If $m=-1/2, -1/4, -3/4, \ldots, -\frac{[2p-1]_{p=1}^{p=q}}{2q}$, $(1+\frac{1}{m})^m$ is imaginary.

(c) If $m=-1/2$, $e<e+\sqrt{(-1)}<\infty$.

(d) If $m=-1$, $\infty<e-\infty<-e$.

Therefore the given statements are proved true unless $0>m\geqq-1$. For case (iv) no general statement of relative magnitudes can be made on account of discontinuous functions.

376. Proposed by W. W. BEMAN, Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.

If $\frac{(1+1/m)^m}{e}=1-a_1\frac{1}{m}+a_2\frac{1}{m^2}-a_3\frac{1}{m^3}+\ldots$, prove $na_n=\sum_{k=1}^{k=n}\frac{k}{k+1}a_{n-k}$, and compute $a_1, a_2, a_3, \ldots, a_8$.

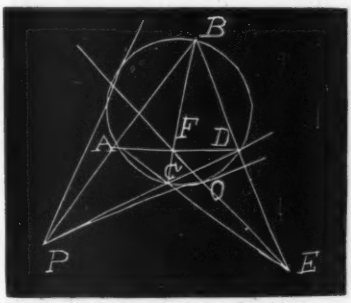
No solution of this problem has been received.

GEOMETRY.

400. Proposed by FRANCIS RUST, C. E., Pittsburgh, Pennsylvania.

Given a circle and a point P without; construct, *using the straight edge only*, the two tangents to the circle through P.

II. Solution by GEORGE W. HARTWELL, Hamline University, St. Paul, Minnesota.



Through P draw any two secants AB and CD, cutting the circle in A, B, C, and D. Join A and D, and A and C; B and D, and B and C. AC and BD meet at E, and AD and BC meet at F. Join E and F. EF is the polar of P. Then the points O and M in which the line EF intersects the circle are the points of tangency.

402. Proposed by H. PRIME, Boston, Mass.

The diameter of a hoop-shaped ring (or collar) is 24 inches at one edge and 28 inches at the other edge. A cross-section is a crescent with circular arcs of 120° and 60°, whose common chord is 4 inches long. Find its volume by elementary methods (without the use of calculus or the center of gravity).

Solution by H. E. TREFETHEN, Colby College.

Denote the given chord by AB, the axis of the ring by QQ, the arc of 120° by s, of 60° by s'. Let ABC be an equilateral triangle. Complete the arcs s and s', and through A and C draw their diameters parallel to QQ.

Then we have $QA=12$, $QC=10$, 4=radius of arc s', $4\sqrt{3}/3$=radius of arc s. $8\pi/3-4\sqrt{3}/3$=area of segment ABs', $16\pi/9-4\sqrt{3}/3$=area of ABs. $v=16.2\sqrt{3}.\pi/6$=volume generated by either segment revolving about its diameter, $2\sqrt{3}$ being the projection of AB on the axis.

Put V and V' for the volumes generated respectively by the segments ABs and ABs' revolving about QQ. Then apply the theorem: If a plane figure exterior to two parallel lines in its plane revolve in succession about each of them as axes, the difference between the volumes of the solids thus generated is equal to the area of the generating figure multiplied by 2π times the distance between the axes. Thus $V-v=(16\pi/9-4\sqrt{3}/3).2\pi.12$, $V'-v=(8\pi/3-4\sqrt{3}/3).2\pi.10$, $V-V'=16\pi(9\sqrt{3}-2\pi)/3=155.9113$ cubic inches= required volume.

PROBLEMS FOR SOLUTION.

ALGEBRA.

380. Proposed by J. K. ELWOOD, Superintendent Lucas Public Schools, Lucas, Kansas.

A and B set out to walk around a cinder path a mile in circumference, and walk 3 hours, A walking 8 miles farther than B. Each reduced his rate one mile per hour at the end of the first hour, and again one mile per hour at the end of the second hour, his speed being otherwise uniform. They start in the same direction, but 12 minutes after A has passed B the third time he turns and walks in the other direction until 6 minutes after he has met B the third time, when he returns to his original direction, and overtakes B four times more. Determine their initial velocities. Illustrate by a time-table.*

381. Proposed by S. A. COREY, Hiteman, Iowa.

A, B, and C simultaneously make assignments of their property for the benefit of their creditors. The assets of A, B, and C were d, e, and q, respectively.

A's indebtedness as principal was a; A's indebtedness as surety for B was g; A's indebtedness as surety for C was h; A's indebtedness as surety for B and C jointly was i, (*i. e.*, B was surety for C, and A, in turn, was surety for B).

B's indebtedness as principal was j; B's indebtedness as surety for A was k; B's indebtedness as surety for C was l.

C's indebtedness as principal was m; C's indebtedness as surety for A was n; C's indebtedness as surety for B was p.

The law requires the surety to pay only such a portion of the debt as his principal cannot pay. What is the amount of the legal indebtness of each?

382. Proposed by C. E. FLANAGAN, Wheeling, West Virginia.

A few days ago I deduced the following formula for finding the value of the unknown quantity in a cubic equation having the form: $x^3+3A^2x=B$.

Let $C=\sqrt{\frac{2B}{A^3}+1}-1$. Then, $x=\frac{2B}{A^2(C^2+12)}+\frac{AC}{4}$.

* This problem is a variation of one by Todhunter. George H. Taber, of Pittsburgh, Pennsylvania, is responsible for it.

It is required to show: (1) How this formula was derived; (2) Under what conditions does it give exact results; (2) In general, what is its degree of approximation; (4) If possible, modify it so that it will always give exact results; (5) If it cannot be so modified show why.

GEOMETRY.

409. Proposed by S. LEFSCHETZ, Ph. D., University of Nebraska.

The sides of a triangle being in arithmetic progression, a and a' being, respectively, the smallest and largest sides; r, R the radii of the inscribed and circumscribed circles. To prove that $6Rr=aa'$.

410. Proposed by A. H. HOLMES, Brunswick, Maine.

Given a focus and two tangents to an ellipse, prove that the locus of the foot of the normal corresponding to either tangent is a straight line.

411. Proposed by C. N. SCHMALL, New York City.

$ABCD$ is a rectangle of known sides. BC being produced indefinitely, it is required to draw a straight line from A cutting CD and BC in X and Y, respectively, so that the intercept XY may be equal to a given straight line.* (Unsolved in *Educational Times.*)

CALCULUS.

331. Proposed by T. H. GRONWALL, Ph. D., C. E., 912 Schiller Building, Chicago, Illinois.

To show that:

$$(1)\ \frac{d^n}{dx^n}\left(\frac{\sin x}{x}\right)=\frac{1}{x^{n+1}}\int_0^x y^n \sin\ (y+\frac{n+1}{2}\pi)\,dy,$$

$$(2)\ \frac{d^n}{dx^n}\left(\frac{1-\cos x}{x}\right)=\frac{1}{x^{n+1}}\int_0^x y^n \sin\ (y+\frac{n}{2}\pi)\,dy.$$

332. Proposed by WILLIAM MARSHALL, Purdue University.

Find the exact or approximate value of the integrals, (a is constant),

$$(1)\ \int_1^\infty \sin(2x+a\log x)\frac{dx}{x^{3/4}};\quad (2)\ \int_1^\infty \cos(2x+a\log x)\frac{dx}{x^{3/4}}.$$

333. Proposed by C. N. SCHMALL, New York City.

Evaluate $\iiint \sqrt{\frac{1-(x^2+y^2+z^2)}{1+x^2+y^2+z^2}}\,dx\,dy\,dz$, where $x^2+y^2+z^2>1$.

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* This problem is No. 16,766 in the *Educational Times.* It was proposed by D. Biddle, who, finding that no solutions of it appeared after a time, gave himself what he calls "An Easy Method of Approximating to a Correct Solution." This appeared in the issue of April 1, 1912, page 180. Mr. Biddle gives a few remarks on the problem in which he says that it is one of the many problems discussed in a paper by Prof. R. C. Archibald, of Brown University, entitled "Discussion and History of Certain Geometrical Problems of Heraclitus and Appollonius." He further says that the solution given there involves curves which are beyond the scope of "ruler and compass." He also refers to a solution by the hyperbola given in the *Educational Times-Reprint*, Volume XX, New Series, page 37. The paper of Professor Archibald was read before the Edinburgh Mathematical Society on June 10, 1910.

My object in proposing the problem here is to show that it permits of a very simple solution by *Euclidean Geometry, i. e.*, by ruler and compasses only. The problem is very important and interesting, and deserves the attention of all geometricians.

NOTES AND NEWS.

The autumn meeting of the Southwestern Section of the American Mathematical Society was held at the University of Kansas on November 29 and 30. That of the San Francisco Section was held at the University of California on October 26. M.

On November 7, Editor Slaught delivered an address before the Iowa State Association of Teachers of Mathematics and Science, at their meeting in DesMoines, on the topic: "The changing conditions in relation to the secondary courses in mathematics." F.

The first number of volume 2 of the *Revista de la Sociedad Matematica Espanola*, which appeared in October, contains a new section devoted to bibliographic notices. Authors and publishers are invited to send copies of their publications for such notices to the Society at San Bernardo, 51, Madrid, Spain. M.

The nineteenth annual meeting of the American Mathematical Society will be held at Cleveland, Ohio, December 31 to January 2, in connection with the annual convocation of the American Association for the Advancement of Science. The winter meeting of the Chicago Section will be merged with the annual meeting at Cleveland. M.

The twelfth annual meeting of the Central Association of Science and Mathematics Teachers was held at Northwestern University, Evanston, Illinois, on Friday and Saturday, November 29 and 30. An important program for mathematics teachers occupied two sessions, on Friday afternoon and Saturday morning. S.

The Illinois Chapter of the Society of the Sigma Xi held its October meeting in honor of M. Henri Poincaré. Professor Borel of Paris spoke and called especial attention to the emphasis which his distinguished colleague had placed on the concepts of invariants and groups. Professor J. B. Shaw read a paper on Poincaré as an investigator. M.

Professor E. Borel, of Paris, France, and Professor Vito Volterra, of Rome, Italy, who were in this country for the purpose of taking part in the dedication of Rice Institute at Houston, Texas, made visits to several universities, including Illinois, Chicago, and Wisconsin in the West, and Columbia in the East. Professor Volterra also attended the meeting of the American Mathematical Society held in New York at the time of his visit there. S.

The Card Section of the Library of Congress expects to have two sets of cards for the great German and French Mathematical Encyclopedias ready for sale during December of the present year. The two sets are known as "author set" and "dictionary set." The latter contains enough copies of

the cards to make all the entries required in a dictionary catalogue and will probably cost about twice as much as the former set. Standing orders for these cards are invited at two cents for one copy of each card, and seven-tenths of a cent for each after the first entry. These cards will doubtless make the great mathematical encyclopedias very much more useful, especially since the regular indices to many of the volumes cannot appear for a number of years. M.

The readers of the MONTHLY are referred to a sketch of the life and works of the late Professor Henri Poincaré in *Science* for August 4, 1912, by Professor G. A. Miller. Professor Poincaré died suddenly on July 17. He had expected to attend the International Congress at Cambridge in August, and was to be one of the speakers at the dedication of Rice Institute at Houston, Texas, in October. Other noted mathematicians who gave addresses at this dedication were Professor Emile Borel, of the University of Paris, Professor Carl Störmer, of the University of Christiania, and Professor Vito Volterra, of the University of Rome. Professor Borel also gave lectures at the University of Illinois, the University of Chicago, and the University of Wisconsin.

Beginning in January, 1913, with Volume XX, THE AMERICAN MATHEMATICAL MONTHLY will pass under the control of an Editorial Board consisting of one representative from each of the nine institutions which have contributed toward a subsidy fund for its promotion, together with B. F. Finkel, the founder of the journal.

The contributing institutions are Colorado College, and the Universities of Chicago, Illinois, Missouri, Minnesota, Nebraska, Kansas, Indiana, and Iowa. The editorial representatives are respectively, Florian Cajori, H. E. Slaught, G. A. Miller, E. R. Hedrick, W. H. Bussey, W. C. Brencke, C. H. Ashton, R. D. Carmichael, and A. G. Smith.

It is proposed to make this journal appeal strongly to the great body of teachers of mathematics in the collegiate and advanced secondary fields, being careful not to encroach upon the domain of any existing journals, either above or below. The animating spirit of this reorganization is the conviction that such a journal is needed not only to direct attention to questions of improvement in teaching in these fields, but also to foster the development of the scientific spirit among large numbers who are not now reached by the more highly technical journals.

The Monthly will continue to publish carefully selected scientific articles, and will encourage the production of such articles by students and teachers in the earlier stages of research work, but relatively greater attention than heretofore will be given to pedagogical and historical questions of interest and value to teachers of collegiate mathematics. For instance, an important feature of the new volume beginning in January will be the serial

publication of Professor Cajori's latest research on "The History of the Logarithmic and Exponential Concepts."

Notwithstanding this proposed enlargement of the Monthly and the doubling of its cost of production, due to the employment of the best available service in mathematical printing, the subscription price will remain at Two Dollars per year. This is made possible only by the subsidy contributions mentioned above, and it is done with the hope that the subscription list may speedily grow to such proportions as to make the journal self-supporting.

Renewals and new subscriptions should be made payable to the Treasurer, B. F. Finkel, Springfield, Mo.

Contributed articles and official correspondence should be addressed to the Managing Editor, H. E. Slaught, 5548 Monroe Avenue, Chicago, Illinois, while all problems and solutions should be sent, for the present, to B. F. Finkel, Springfield, Mo.

The Monthly will soon issue an index of Volumes I-XIX, which will render accessible the large number and variety of contributions already published in this journal. Back numbers can be supplied at 25 cents each, and single volumes at two dollars and fifty cents each. No complete sets of the MONTHLY are available, many of the numbers being out of print. All correspondence relating to back numbers should be directed to B. F. Finkel. S.

Having gotten behind one number, we are obliged to combine two numbers in one that we may get the December number out on time and thus have the way clear for the January number, which is to come out under the new organization. F.

BOOKS.

The ABC of the Differential Calculus. By William Dyson Wansbrough, Author of the "Portable Steam Engine: Its Construction and Management;" The Proportions and Movement of Slide Valves;" "Modern Steam Boilers;" etc. Third Edition. 16mo. Cloth, xii+148 pages. Price, $1.50. New York: D. VanNostrand Co.

The author attempts to give the beginner an understanding of the Calculus by beginning with its most simple concepts. His illustrations are so simple and the details carried out so fully that any one ambitious enough to desire an insight into one of the most interesting of all mathematical subjects can readily gain such insight by perusing this book. F.

A Manual of Laboratory Exercises in Physics. By Frederick R. Gorton, B. S., M. A., Ph. D., Professor of Physics, Michigan State Normal College. 8vo. Cloth, xv+166 pages. New York: D. Appleton & Co.

In this little book is set forth a number of simple experiments in mechanics, sound, heat, light, and electricity and magnetism. F.

Differential and Integral Calculus. An Introductory Course for Colleges and Engineering Schools. By Lorrain S. Hulburt, Collegiate Professor of Mathematics in the Johns Hopkins University. 8vo. Cloth, xviii+481 pages. New York: Longmans, Green & Co.

This volume of the Calculus is divided into six books, as follows: Book I, treats of the differential calculus; Book II, of the integral calculus; Book III, introduction to analytical geometry of three dimensions, Book IV, functions of more than one argument; Book V, theorems of Taylor and MacLaurin, integration of rational fractions, and envelopes; Book VI, an introduction to ordinary differential equations.

The various topics of the Calculus discussed in this book are treated with great clearness and without undue complexity of notation; the problems have been carefully selected, some of them from the older texts and many of them have been created by the author himself. The publishers have been very fortunate in presenting the book in a very attractive style as to type and diagrams. F.

Daytime and Evening Exercises in Astronomy. For Schools and Colleges. By Sarah Francis Whiting, Sc. D., Whitin Observatory, Wellesley College. 8vo. Cloth, xiv+104 pages. New York, Boston, and Chicago: Ginn & Co.

It is hoped that this little book will stimulate an interest in the study of astronomy, and that we shall again find a large number of students in our colleges demanding courses in this oldest and one of the most inspiring of the sciences. F.

New Analytical Geometry. By Percy F. Smith, Professor of Mathematics in the Sheffield Scientific School of Yale University, and Arthur Sullivan Gale, Ph. D., Professor of Mathematics in the University of Rochester. 8vo. Cloth, x+342 pages. New York, Boston, and Chicago: Ginn & Co.

This book holds an intermediate place between the authors' *Introduction to Analytical Geometry* and the *Elements of Geometry.* The method of treatment is the same as that pursued in the two previous books. Those teachers who have found the *Elements* too exhaustive and the *Introduction* too limited will be glad to have this book, which will satisfy them in every particular. F.

Plane Geometry. By William Belz, A. M., Vice-Principal and Head of the Department of Mathematics in the East High School, Rochester, New York, and Harrison E. Webb, A. B., Head of the Department of Mathematics in the Central Commercial and Manual Training High School, Newark, New Jersey, with the editorial coöperation of Percy F. Smith, Professor of Mathematics in the Sheffield Scientific School of Yale University. 8vo. Cloth, x+332 pages. Price, $1.00. Boston, New York, and Chicago: Ginn & Co.

The authors' apology for adding another text to the long list of existing geometrical text-books is to effect a compromise between the extreme demands of certain ultra progressives and the equally untenable position of the stand-patters. To this end they have written this text with the following features: (1) A preliminary course followed by the demonstrative course; (2) The demonstrative course built up not only in a topical, but also in a psychological order; (3) Methods embodied in the text aiming to make pupils independent of the printed page; (4) Equal consideration to various types of exercises; (5) An extensive but not excessive list of applied problems; and (6) Providing a minor and a major course. A student who has the mastery of this text will be well equipped for the practical problems which he may meet in life. F.

Elements of Plane Trigonometry. High School Edition. By Robert E. Moritz, Ph. D. (Nebraska), Ph. N. D. (Strassburg), Professor of Mathematics, University of Washington. Small 8vo. Cloth, viii+315 pages. Price, $1.00. New York: John Wiley & Sons.

This book, we are told, is an effort on the part of the author to bring about a more perfect adjustment of the teaching of trigonometry with the teaching of the subjects on which it rests, and with the progress of the arts and sciences to which it applies. The book justifies the author's claim. F.

A History of the Theories of Aether and Electricity from the Age of DesCartes to the Close of the Nineteenth Century. By E. T. Whittaker, Hon. Sc. D. (Dublin); F. R. S.; Royal Astronomer of Ireland. New York and London, England: Longmans, Green & Co. 8vo. Cloth, xix+475 pages. Price, $3.00.

The scope of this splendid work may be inferred from its table of contents. The first chapter treats of the theory of the æther in the seventeenth century; chapter II, Electric and magnetic science, prior to the Introduction of the Potentials; chapter III, Galvanism, from Galvani to Ohm; chapter IV, The luminiferous medium, from Bradley to Fresnel; chapter V, The æther as an elastic solid; chapter VI, Faraday; chapter VII, The mathematical electricians of the middle of the nineteenth century; chapter VIII, Maxwell; chapter IX, Models of the æther; chapter X, The followers of Maxwell; chapter XI, Conduction in solutions and gases, from Faraday to J. J. Thompson; chapter XII, The theory of æther and electrons in the closing years of the nineteenth century.

In the treatment of some of the subjects, vector analysis is used. To make the book readable to those unfamiliar with vector analysis, the author has introduced at the beginning a page of vector notation. This work will be of inestimable value to the teacher of physics. It was prepared by an able and eminent writer on physical and mathematical subjects, a fact insuring reliability and accuracy in statement and discussion. F.

Laboratory Studies in Chemistry. By Robert H. Bradbury, A. M., Ph. D., Head of the Department of Science in the Southern High School, Philadelphia. 8vo. Cloth, ix+129 pages. New York and Chicago: D. Appleton & Co.

This manual covers the various syllabi which teachers preparing students for college have to consider, and requires simple apparatus and generally inexpensive material. F.

A Shorter Geometry. By C. Godfrey, M. V. O., M. A., Head Master of the Royal Naval College, Osborne; formerly Senior Mathematical Master at Winchester College; and A. W. Siddons, M. A., late Fellow of Jesus College, Cambridge; Assistant Master at Harrow School. 8vo. Cloth, xxii+301 pages. Price, 80 cents. Cambridge: The University Press. G. P. Putnam's Sons, American Agents.

The plan of this book presents the subject in three stages, as follows: First, introductory work concerned with fundamental concepts; Second, discovery of fundamental facts of geometry, by experiment and intuition; and, Third, subsequent deductive development of the propositions. The book is beautifully printed on good paper, and handsomely bound. The authors are neither ultra-radical nor ultra-conservative. They clearly recognize the weakness of many of the modern methods as well as those of former times. The book is pedagogically wholesome. F.

Mathematical Wrinkles. For Teachers and Private Learners. Consisting of Knotty Problems; Mathematical Recreations, Answers and Solutions; Rules of Mensuration; Short Methods; Helps, Tables, Etc. By Sam I. Jones, Professor of Mathematics in the Gunter Biblical and Literary College, Gunter, Texas. 12mo. Half Leather, viii+321 pages. Price, $1.65.

This book ought to be in the library of every teacher who has to teach any of the mathematical subjects in high schools, academies, or country schools. It contains a vast amount of useful and interesting material which such teachers ought to know, and with which they can often arouse interest and stimulate enthusiasm in their classes. It is a very regrettable fact that the larger per cent of our teachers who teach mathematics have little vital interest in the subject, and this fact is manifested by the extreme poverty of their libraries, not only in mathematics but in other subjects as well. Teachers who have not enough interest in their subjects to keep themselves informed on what is going on in this rapidly progressing world, had better give their places to others who have a higher ambition than merely making a living. Let the teacher of elementary mathematics buy a copy of this book. It will do you good. It contains problems, solutions, quotations from mathematicians, puzzles, etc. F.

Practical Descriptive Geometry. By William Griswold Smith, M. E., Assistant Professor of Descriptive Geometry and Kinematics, Armour Institute of Technology. 8vo. Cloth, ix+208 pages. New York: McGraw-Hill Book Company.

The author of this book, while recognizing the indisputable excellence of many of the text-books on Descriptive Geometry, feels that many of these excellent treatises are only useful as reference books; others are incomplete in essentials; still others are faddish, emphasizing certain features and treating others inadequately; while even the best convey only a slight idea of the practical value of the subject. Keeping these defects before him, we believe he has written a book that is accurate, practical, and teachable. F.

Essentials of Calculus. By E. J. Townsend, Ph. D. (Göttingen), Professor of Mathematics, University of Illinois, and G. A. Goodenough, M. E. (Illinois), Associate Professor of Mechanical Engineering, Univsrsity of Illinois. 8vo. Cloth, x+355 pages. Chicago: Henry Holt & Co.

The average college student and students of technical schools will find their needs quite well met in this book. The work is based on the theory of limits, and the usual division of the subject into differential and integral has been pretty generally disregarded. This text will meet the demands of a large number of teachers in our colleges and technical schools. F.

Gravitation. By Frank Harris, B. A. (Oxon), Late Executive Engineer, and Associate M. Inst. C. E. 8vo., xi+107 pages. Price, $1.00. New York: Longmans, Green & Co.

The author presents, in this volume, a theory as to the nature of gravitation. He begins his discussion with an inquiry concerning the motion of a massless spherical shell in an incompressible, frictionless fluid of unit density. He next considers the motion of two spheres in an infinite fluid. His discussion, in which he uses mathematical reasoning freely, is interesting and instructive. F.

Complete School Algebra. By Herbert E. Hawkes, Ph. D., Professor of Mathematics in Columbia University; William A. Luby, A. B., Head of the Department of Mathematics, Central High School, Kansas City, Mo.,

and Frank C. Tuton, Ph. B., Principal of Central High School, St. Joseph, Mo. 12mo Cloth, x+507 pages. Price, $1.25. Boston and Chicago: Ginn & Co.

"The 'Complete School Algebra,' which includes between the covers of a tingle volvme—with the necessary adaptation and abridgment—all the material of the authors' 'First Course in Algebra' and 'Second Course in Algebra,' is designed for those schools which find a one-book course best suited to their needs.

"The first twenty-three chapters contain the greater portion of the work usually taken up during the first year. Then follows the review material, each topic being given a broader and more advanced treatment than is permissible in first year work. New matter is used throughout, and many new applications are given in order to make a fresh and inviting appeal to the student. In the remaining chapters those advanced topics considered necessary by the best secondary schools are included.

The Hawkes, Luby, and Touton Algebras are marked by freshness and sanity of method. Wealth of illustrative material, correlation with arithmetic, geometry, and physics, prominence given the equation, emphasis on checking, and extensive work with graphs are a few of the features. These algebras—in a one-book and a two-book series—offer an arrangement of ideal flexibility in their adaptation to the needs of different schools."

Lectures on the Theory of Functions of Real Variables. Volume II. By James Pierpont, LL. D., Professor of Mathematics in Yale University. 8vo. Cloth, xiii+645 pages. Price, $5.00.

In the compilation of this second volume, the author has rendered the American mathematician an invaluable service and has added to American Mathematical literature a most creditable and monumental work

The present volume comprises seventeen chapters treating the following subjects in order: Point Sets and Proper Integrals; Improper Multiple Integrals; Series; Multiple Series; Series of Functions; Power Series; Infinite Products; Aggregates; Ordinal Numbers; Point Sets, Measure; Lebesgue Integrals; Fourier's Series; Discontinuous Functions; Derivatives, Extremes, Variation; Sub- and Infra-Uniform Convergence; and Geometric Notions.

Pierpont's two volumes together with Hobson's treatise on the same subject present a fairly complete discussion of all that is known on the Theory of Functions of a Real Variable at the present time.

The publishers are to be congratulated for the very excellent style of printing and binding adopted in this work. F.

Syllabus of Mathematics. A Symposium compiled on the Teaching of Mathematics to Students of Engineering. Accepted by the Society for the Promotion of Engineering Education at the Nineteenth Annual Meeting held at Pittsburgh, Pennsylvania, June, 1911. 8vo. Cloth, 136 pages. Price, 75 cents. Ithaca: Office of the Secretary.

The Society feels that this report should be of value to teachers of mathematics in showing them what are considered fuudamentals for engineering subjects and to indicate to teachers of engineering the preparation which they may reasonably expect their students to have had. It should also serve as a reference syllabus for students or engineers who wish to systematize their knowledge of mathematics. F.

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THE AMERICAN MATHEMATICAL MONTHLY.

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VOL. XIX. DECEMBER, 1912. NO. 12.

RETROSPECT AND PROSPECT.

By H. E. SLAUGHT, The University of Chicago.

With this issue THE AMERICAN MATHEMATICAL MONTHLY closes the nineteenth year of its history. It was founded in 1894 by Benjamin F. Finkel, then professor of mathematics at Kidder Institute, Kidder, Missouri. Associated with him in the editorship was J. M. Colaw; then principal of the high school at Monterey, Virginia. Other associate editors who have assisted from time to time are Saul Epsteen, during the years 1904-5 and Oliver E. Glenn during 1906.

Professor Finkel, as author of the well known Mathematical Solution Book, naturally gave great impetus, through the columns of his newly established journal to the proposal and solution of problems. The number and character of the contributors to these departments and the sustained interest in these contributions even up to the present time, on the part of the readers of this journal are of no small import in its history. But appropriate space has also been given to contributed articles of a widely diversified character, including biographies of mathematicians, historical, pedagogical, and bibliographical papers, and elementary research in pure and applied mathematics. The names of the contributors constitute a long and worthy list, as will be evident from the general index to volumes I to XIX soon to be published. These contributions have not only provided interest and profit for the constituency of the Monthly, but, in not a few cases, they have been veritable stepping stones on which the authors themselves have risen to greater mastery of technical research in the more advanced phases of mathematics.

It is in nowise the purpose of this article to relate the history of the Monthly in detail, in fact this could hardly be done with completeness except by the founder himself and those who were intimately associated with him during the early years. Suffice it to say that the years 1894-1902 constitute an epoch in the history during which the full financial and editorial responsibility rested upon the founder alone, while in the period 1903-1912, the full editorial responsibility, aside from the problem departments, rested upon others, and some financial assistance was received from outside sources.